University of Toronto

Final Exam

Date — Dec 19, 2019: 2pm

Duration — 2.5 hrs

ECE 331 — Analog Electronics

Lecturer — D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

- Equation sheet is on the last page of this test.
- Unless otherwise stated, assume $g_m r_o \gg 1$
- Notation: 1.5e-3 is equivalent to 1.5×10^{-3}
- Non-programmable calculator is allowed; No other aids are allowed
- Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Last Name: ________________________________

First Name: ________________________________

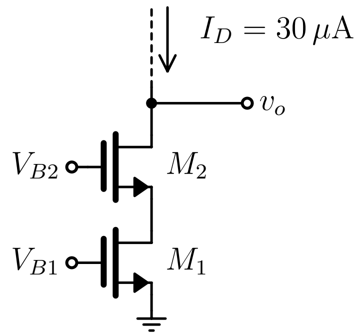
Student #: ________________________________

Question	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
Total:	36	

Grading Table
(do not write in above table)

[6] **Q1.** Consider the wide-swing current mirror shown below where the desired output current is 30 μA. Given that M_1 and M_2 are identical in size and the minimum output voltage is 0.5 V, find the length of the transistors such that the current mirror output resistance is 60 MΩ.

NMOS: $V_{tn} = 0.3\,\text{V}; \mu_n C_{ox} = 160\,\mu\text{A/V}^2; \lambda'_n = 40\,\text{nm/V}$



Solution

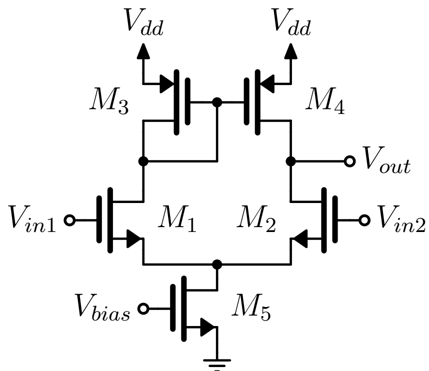
$V_{o,min} = 0.5\,\text{V} = 2V_{ov}$; $V_{ov} = V_{o,min}/2 = 0.25\,\text{V}$ for both transistors
$g_m = 2 * I_D/V_{ov} = 2 * (30e-6)/(0.25) = 240\,\mu\text{A/V}$ for each transistor
$R_{out} \approx g_m r_o^2$; $r_o = \sqrt{R_{out}/g_m} = \sqrt{(60e6)/(240e-6)} = 500\,\text{k}\Omega$
$r_o = L/(|\lambda'_n| I_D)$
$L = r_o * |\lambda'_n| * I_D = (500e3) * |(40e-9)| * (30e-6) = 600\,\text{nm}$
$L = 600\,\text{nm}$

Answer

L = 600 nm

Q2. Consider the differential to single ended amplifier shown below. All transistor lengths are 200 nm and have $V_{ov} = 0.15\,\text{V}$. Also, $I_{D5} = 80\,\mu\text{A}$ and $V_{dd} = 1.8\,\text{V}$.

NMOS: $V_{tn} = 0.25\,\text{V}$;$\mu_n C_{ox} = 200\,\mu\text{A/V}^2$;$\lambda'_n = 50\,\text{nm/V}$
PMOS: $V_{tp} = -0.3\,\text{V}$;$\mu_p C_{ox} = 60\,\mu\text{A/V}^2$;$\lambda'_p = -40\,\text{nm/V}$



[3] (a) Find the small-signal gain v_{out}/v_{id} where $v_{id} \equiv v_{in2} - v_{in1}$

Solution

All $L_i = 200\,\text{nm}$

Since $I_{D5} = 80\,\mu\text{A}$, I_{D1} to I_D4 all equal $40\,\mu\text{A}$

$r_{o2} = L_2/(|\lambda'_n| * I_{D2}) = (200e-9)/(|(50e-9)| * (40e-6)) = 100\,\text{k}\Omega$

$r_{o4} = L_4/(|\lambda'_p| * I_{D4}) = (200e-9)/(|(-40e-9)| * (40e-6)) = 125\,\text{k}\Omega$

$R_{out} = r_{o2}||r_{o4} = (100e3)||(125e3) = 55.56\,\text{k}\Omega$

$g_{m2} = 2 * I_{D2}/V_{ov2} = 2 * (40e-6)/(0.15) = 533.3\,\mu\text{A/V}$

$v_{out}/v_{id} = -g_{m2} * R_{out} = -(533.3e-6) * (55.56e3) = -29.63\,\text{V/V}$

$v_{out}/v_{id} = -29.63\,\text{V/V}$

Answer

$v_{out}/v_{id} = -29.63\,\text{V/V}$

[3] (b) Find the maximum ($V_{cm,max}$) and minimum ($V_{cm,min}$) common-mode input voltage that keep all transistors in the active region.

Solution

Given: $V_{ov,i} = 0.15$ for all i

$V_{cm,min} = V_{ov5} + V_{tn} + V_{ov2} = (0.15) + (0.25) + (0.15) = 0.55\,\text{V}$

$V_{cm,min} = 0.55\,\text{V}$

For $V_{cm,max}$, first find the bias voltage for V_{D1}

$V_{D1} = V_{dd} - |V_{tp}| - V_{ov3} = (1.8) - |(-0.3)| - (0.15) = 1.35\,\text{V}$

V_{in1} can go up to V_{tn} above V_{D1} so

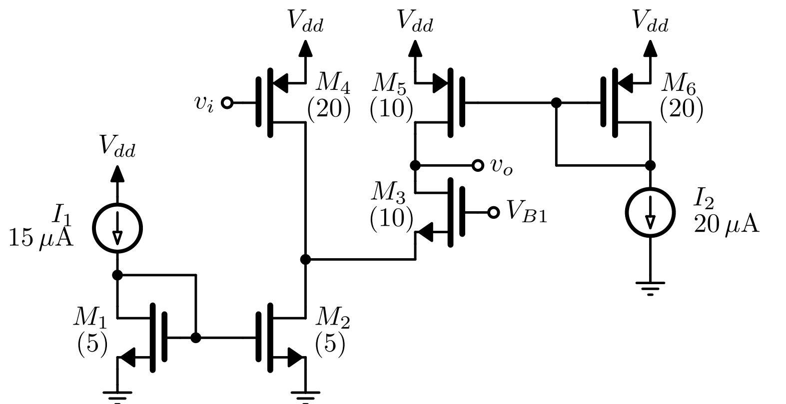
$V_{cm,max} = V_{D1} + V_{tn} = 1.6\,\text{V}$

Answer

$V_{cm,min} = 0.55\,\text{V}$; $V_{cm,max} = 1.6\,\text{V}$

Q3. Consider the amplifier shown below where all transistor lengths are 180 nm. The bracketed numbers are each of the transistor's (W/L) value.

NMOS: $V_{tn} = 0.25\,\text{V}$; $\mu_n C_{ox} = 200\,\mu\text{A/V}^2$; $\lambda'_n = 50\,\text{nm/V}$
PMOS: $V_{tp} = -0.3\,\text{V}$; $\mu_p C_{ox} = 60\,\mu\text{A/V}^2$; $\lambda'_p = -40\,\text{nm/V}$



[3] (a) Calculate the drain currents, overdrive voltages and r_o for $M_2/M_3/M_4/M_5$ transistors.

Solution

$I_{D1} = I_1 = 15\,\mu\text{A}$; $I_{D2} = (W/L)_2/(W/L)_1 * I_{D1} = (5)/(5) * (15e-6) = 15\,\mu\text{A}$;
$I_{D6} = I_2 = 20\,\mu\text{A}$; $I_{D5} = (W/L)_5/(W/L)_6 * I_{D6} = (10)/(20) * (20e-6) = 10\,\mu\text{A}$;
$I_{D3} = I_{D5} = 10\,\mu\text{A}$; $I_{D4} = I_{D2} - I_{D3} = (15e-6) - (10e-6) = 5\,\mu\text{A}$
$I_{D2} = 15\,\mu\text{A}$; $I_{D3} = 10\,\mu\text{A}$; $I_{D4} = 5\,\mu\text{A}$; $I_{D5} = 10\,\mu\text{A}$;
$\lambda_n = \lambda'_n/L = (50e-9)/(180e-9) = 0.2778\,\text{V}^{-1}$;
$r_{o2} = 1/(\lambda_n * I_{D2}) = 240\,\text{k}\Omega$; $r_{o3} = 1/(\lambda_n * I_{D3}) = 360\,\text{k}\Omega$
$\lambda_p = \lambda'_p/L = (-40e-9)/(180e-9) = -0.2222\,\text{V}^{-1}$;
$r_{o4} = 1/(|\lambda_p| * I_{D4}) = 900\,\text{k}\Omega$; $r_{o5} = 1/(|\lambda_p| * I_{D5}) = 450\,\text{k}\Omega$
$r_{o2} = 240\,\text{k}\Omega$; $r_{o3} = 360\,\text{k}\Omega$; $r_{o4} = 900\,\text{k}\Omega$; $r_{o5} = 450\,\text{k}\Omega$;
For nmos: $V_{ov} = \sqrt{(2I_D)/(\mu_n C_{ox}(W/L))} = 100\sqrt{I_D/(W/L)}$
$V_{ov2} = 0.1732\,\text{V}$; $V_{ov3} = 0.1\,\text{V}$
For pmos: $V_{ov} = \sqrt{(2I_D)/(\mu_p C_{ox}(W/L))} = 182.6\sqrt{I_D/(W/L)}$
$V_{ov4} = 91.29\,\text{mV}$; $V_{ov5} = 0.1826\,\text{V}$

Answer

$I_{D2} = 15\,\mu\text{A}$; $I_{D3} = 10\,\mu\text{A}$; $I_{D4} = 5\,\mu\text{A}$; $I_{D5} = 10\,\mu\text{A}$;
$V_{ov2} = 0.1732\,\text{V}$; $V_{ov3} = 0.1\,\text{V}$; $V_{ov4} = 91.29\,\text{mV}$; $V_{ov5} = 0.1826\,\text{V}$;
$r_{o2} = 240\,\text{k}\Omega$; $r_{o3} = 360\,\text{k}\Omega$; $r_{o4} = 900\,\text{k}\Omega$; $r_{o5} = 450\,\text{k}\Omega$;

[3] (b) Estimate the small-signal gain v_o/v_i
Hint: The impedance looking into the drain of M_3 is much higher than r_{o5} so it can be ignored

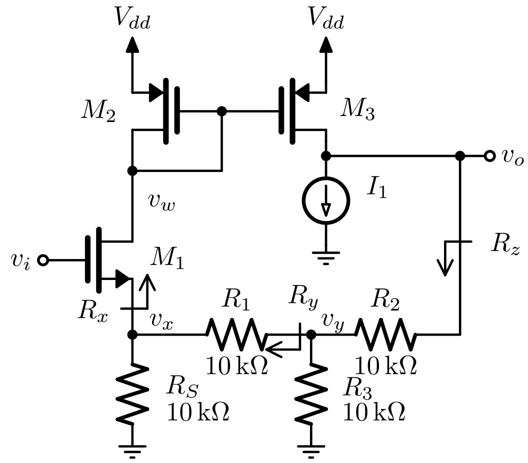
Solution

$g_{m4} = (2 * I_{D4})/V_{ov4} = 109.5\,\mu\text{A/V}$
$I_{sc} = g_{m4} v_i$ and $R_{out} \approx r_{o5}$
$v_o = -I_{sc} R_{out} \Rightarrow v_o/v_i = -g_{m4} * r_{o5} = -49.3\,\text{V/V}$
$v_o/v_i = -49.3\,\text{V/V}$

Answer

$v_o/v_i = -49.3\,\text{V/V}$

[6] **Q4.** Consider the feedback amp shown below with the input signal, v_i. All current sources are ideal (infinite output resistance).



all $r_o \to \infty$
$g_{m1} = 100\,\mu\text{A/V}$
$g_{m2} = 100\,\mu\text{A/V}$
$g_{m3} = 5\,\text{mA/V}$

Using loop analysis, find L, A_∞, and v_o/v_i. (Assume $d = 0$)

Solution

Using the definitions of R_x, R_y, R_z and nodes v_w, v_x, v_y show above,

$R_x = 1/g_{m1} = 10\,\text{k}\Omega$; $R_y = R_x||R_S + R_1 = 15\,\text{k}\Omega$
$R_z = (R_y||R_3) + R_2 = 16\,\text{k}\Omega$
Also, since $r_o3 \to \infty$ and the current mirror is ideal,
$R_o = R_z = 16\,\text{k}\Omega$

Breaking the loop at the gate of M_3, we have
$v_o/v_{g3} = -g_{m3} * R_o = -(5e-3) * (16e3) = -80\,\text{V/V}$
$v_y/v_o = (R_y||R_3)/((R_y||R_3) + R_2) = ((15e3)||(10e3))/(((15e3)||(10e3)) + (10e3)) = 0.375\,\text{V/V}$
$v_x/v_y = (R_x||R_S)/((R_x||R_S) + R_1) = ((10e3)||(10e3))/(((10e3)||(10e3)) + (10e3)) = 0.3333\,\text{V/V}$
$v_w/v_x = g_{m1}/g_{m2} = (100e-6)/(100e-6) = 1\,\text{V/V}$
$L = -v_o/v_{g3} * v_y/v_o * v_x/v_y * v_w/v_x = -(-80) * (0.375) * (0.3333) * (1) = 10$

$L = 10$

For A_∞, $L \to \infty$ which results in $v_w \to 0$. Since $v_w = 0$, the small signal current $i_{D1} = 0$ (meaning the current is constant) resulting in the small-signal voltages $v_x = v_i$ and no small-signal current flows into the source of M_1.

As a result, $v_y/v_i = 1 + R_1/R_S = 1 + (10e3)/(10e3) = 2\,\text{V/V}$
and $v_o/v_y = 1 + R_2/(R_3||(R_1 + R_S)) = 1 + (10e3)/((10e3)||((10e3) + (10e3))) = 2.5\,\text{V/V}$
Resulting in $A_\infty = v_o/v_y * v_y/v_i = (2.5) * (2) = 5\,\text{V/V}$

$A_\infty = 5\,\text{V/V}$

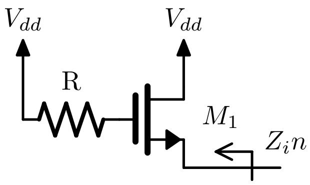
Finally, $v_o/v_i = A_\infty * (L/(1 + L)) = (5) * ((10)/(1 + (10))) = 4.545\,\text{V/V}$

$v_o/v_i = 4.545\,\text{V/V}$

Answer

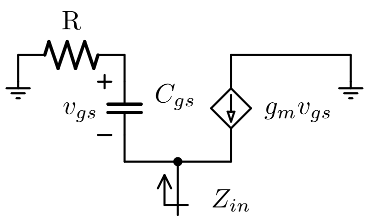
$A_\infty = 5\,\text{V/V}$; $v_o/v_i = 4.545\,\text{V/V}$

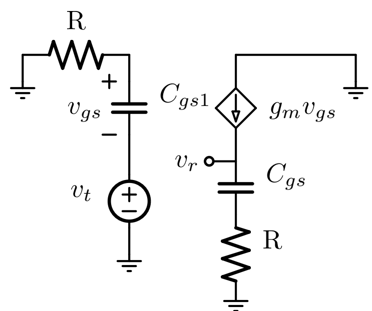
Q5. Consider the input impedance looking into the source of the circuit below. Let $r_o \to \infty$ and for inherent capacitors, only consider capacitance, C_{gs}.



[3] (a) Draw the small-signal model and find equations for the loop gains, L_O and L_S with respect to the port impedance Z_{in}.

Solution





$L_O \equiv -v_r/v_t = -(v_{gs}/v_t)(v_r/v_{gs}) = -\left(\frac{-1/sC_{gs}}{(1/sC_{gs})+R}\right)(g_m((1/sC_{gs})+R)) = \frac{g_m}{sC_{gs}}$

$L_O = \frac{g_m}{sC_{gs}}$

L_S equal zero since the port is shorted and results in zeroing the loop gain.

$L_S = 0$

Answer

$L_O = \frac{g_m}{sC_{gs}}$; $L_S = 0$

[3] (b) Using the loop gains found above, find the equation for Z_{in} and find equations for the low freq impedance, the high freq impedance, and the pole and zero frequencies (in rad/s).

$Z_{P0} = R + (1/sC_{gs}) = \frac{1+sC_{gs}R}{sC_{gs}}$

$Z_{in} = Z_{P0}\left(\frac{1+L_S}{1+L_O}\right) = \left(\frac{1+sC_{gs}R}{sC_{gs}}\right)\left(\frac{1}{1+g_m/sC_{gs}}\right) = \left(\frac{1+sC_{gs}R}{sC_{gs}}\right)\left(\frac{sC_{gs}}{sC_{gs}+g_m}\right)$

$Z_{in} = \frac{1+sC_{gs}R}{g_m+sC_{gs}}$

The low freq impedance is $1/g_m$; The high freq impedance is R ; The pole freq is g_m/C_{gs} rad/s ; The zero freq is $1/C_{gs}R$ rad/s

Answer

$Z_{in} = \frac{1+sC_{gs}R}{g_m+sC_{gs}}$

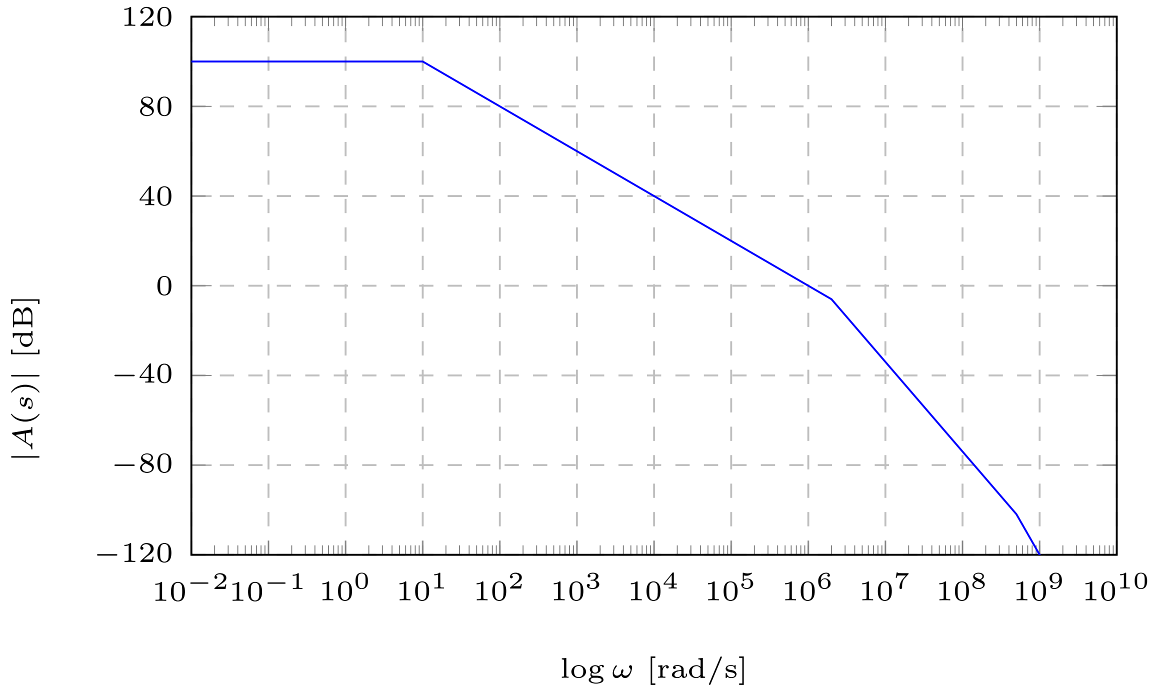
The low freq impedance is $1/g_m$; The high freq impedance is R; The pole freq is g_m/C_{gs} rad/s; The zero freq is $1/C_{gs}R$ rad/s

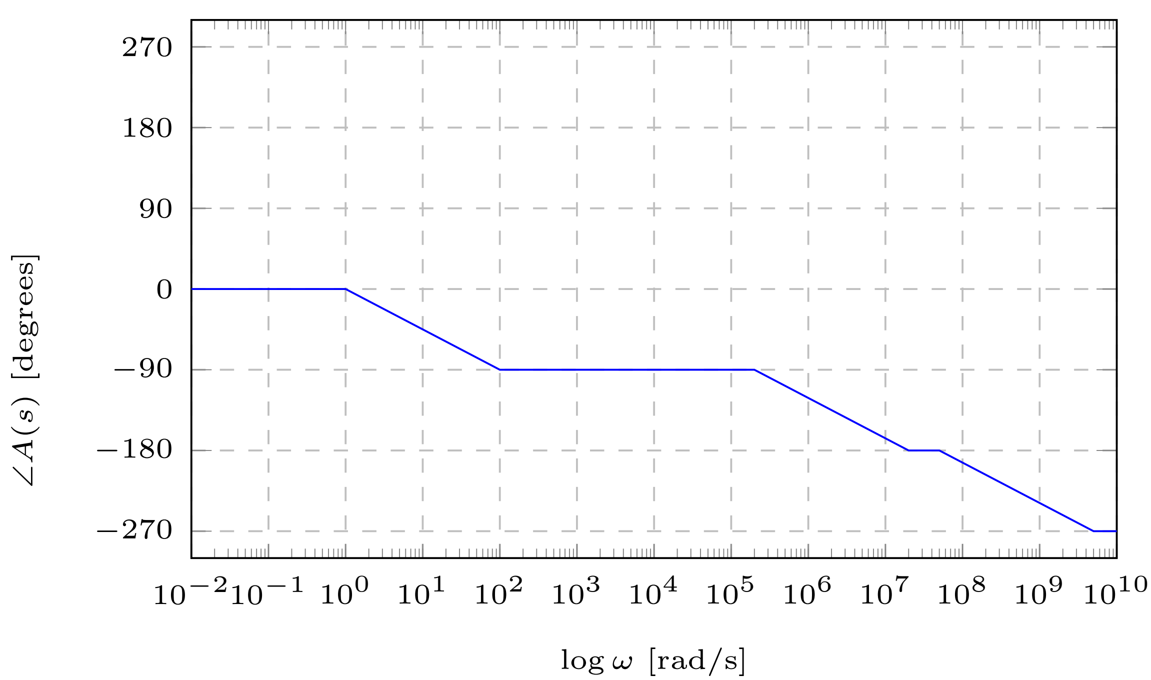
Q6. Assume an opamp is ideal but has the following open-loop gain.

$A(s) = \frac{k_{dc}}{(1+s/\omega_{p1})(1+s/\omega_{p2})(1+s/\omega_{p3})}$ where

$k_{dc} = 100\,\text{k}$, $\omega_{p1} = 10\,\text{rad/s}$, $\omega_{p2} = 2\,\text{Mrad/s}$, and $\omega_{p3} = 500\,\text{Mrad/s}$

[3] (a) Draw the Bode plot for the above loop gain.





Solution

See above graphs

For magnitude response, dc gain extends until $\omega_{p1} = 10\,\text{rad/s}$, then gain drops 20dB/dec until $\omega_{p2} = 2\,\text{Mrad/s}$.
After ω_{p2} the gain drops by 40dB/dec until $\omega_{p3} = 500\,\text{Mrad/s}$

For phase response, phase starts a $0°$ at dc and extends until $\omega_{p1}/10$. Phase then drops to $-90°$ at $\omega_{p1} * 10$. The phase remains at $-90°$ until $\omega_{p2}/10$ where it drops to $-180°$ by $\omega_{p2} * 10$. A similar drop of $-90°$ occurs around ω_{p3}

Answer

[3] (b) Estimate the phase-margin (PM) if the above opamp is used to create a gain of +3 using 2 resistors (a non-inverting configuration) (Hint: Note that the unity gain freq is much greater than ω_{p1} and much less than ω_{p3}.)

Solution

For a non-inverting opamp gain of $K = 3\,\text{V/V}$, $\beta = 1/K = 0.3333$, resulting in the loop gain equal to

$L(s) = \beta A(s) = \frac{L_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})(1+s/\omega_{p3})}$

where $L_0 = \beta * k_{dc} = (0.3333) * (100e3) = 33.33\,\text{k}$ and $\omega_{p1}, \omega_{p2}, \omega_{p3}$ are given above.

For frequencies near ω_t where $\omega_{p1} \ll \omega_t \ll \omega_{p3}$, we can approximate $L(s)$ as

$L(s) \approx \frac{L_0}{(s/\omega p1)(1+s/\omega p2)}$ and making use of $|L(j\omega_t)|^2 = 1$, we have

$\frac{L_0^2}{(\omega_1/\omega_{p1})^2(1+(\omega_t/\omega_{p2})^2)} = 1$ which can be rewritten as $\frac{\omega_{p1}^2\omega_{p2}^2 L_0^2}{\omega_t^2(\omega_t^2+\omega_{p2}^2)} = 1$

This equation can be written as a quadratic equation: $(\omega_t^2)^2 + \omega_{p2}^2(\omega_t^2) - \omega_{p1}^2\omega_{p2}^2 L_0^2 = 0$.

Putting in values, we have $(\omega_t^2)^2 + 4e12\omega_t^2 - 444.4e21 = 0$

Solving for ω_t^2, (it is a quadratic equation) we have 2 solutions of which one is positive (so we keep the positive solution) and therefore $\omega_t^2 = 108.2\,\text{G}$ resulting in $\omega_t = \sqrt{\omega_t^2} = 328.9\,\text{krad/s}$

We can now find the phase of $L(\omega_t)$ as

$\angle L(j\omega_t) = -90 - \text{atan}(\omega_t/\omega_{p2}) = -90 - \text{atan}((328.9e3)/(2e6)) = -99.34\,°$

Finally, the phase-margin (PM) can be found as

$PM = \angle L(j\omega_t) + 180 = (-99.34) + 180 = 80.66\,°$

$PM = 80.66\,°$

Answer

$PM = 80.66\,°$

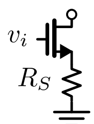
Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $q = 1.602 \times 10^{-19}\,\mathrm{C}$; $V_T = kT/q \approx 26\mathrm{mV}$ at $300\,\mathrm{K}$; $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$; $\omega = 2\pi f$

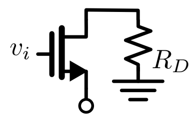
NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $V_{ov} = V_{GS} - V_{tn}$
(triode) $v_{DS} \leq V_{ov}$; $v_D < v_G - V_{tn}$; $i_D = k_n(V_{ov}v_{DS} - (v_{DS}^2/2))$
(active) $v_{DS} \geq V_{ov}$; $i_D = 0.5k_nV_{ov}^2(1+\lambda v_{DS})$; $g_m = k_nV_{ov} = 2I_D/V_{ov} = \sqrt{2k_nI_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $V_{ov} = V_{SG} - |V_{tp}|$
(triode) $v_{SD} \leq V_{ov}$; $v_D > v_G + |V_{tp}|$; $i_D = k_p(V_{ov}v_{SD} - (v_{SD}^2/2))$
(active) $v_{SD} \geq V_{ov}$; $i_D = 0.5k_pV_{ov}^2(1+|\lambda|v_{SD})$; $g_m = k_pV_{ov} = 2I_D/V_{ov} = \sqrt{2k_pI_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

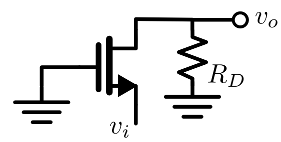
BJT: (active) $i_C = I_Se^{(v_{BE}/V_T)}(1+(v_{CE}/V_A))$; $g_m = \alpha/r_e = I_C/V_T$; $r_e = V_T/I_E$; $r_\pi = \beta/g_m$; $r_o = |V_A|/I_C$; $i_C = \beta i_B$; $i_E = (\beta+1)i_B$; $\alpha = \beta/(\beta+1)$; $i_C = \alpha i_E$; $R_b = (\beta+1)(r_e+R_E)$; $R_e = (R_B+r_\pi)/(\beta+1)$



$i_{sc} \approx -(1/g_m + R_S)^{-1}v_i$
$R_x \approx (1+g_mR_S)r_o$
(Approx due to $g_mr_o \gg 1$)



$v_{oc} \approx v_i$
$R_x \approx 1/g_m + R_D/(g_mr_o)$



$v_o/v_i \approx g_m(r_o||R_D)$

Diff Pair: $A_d = g_mR_D$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta R_D/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta g_m/g_m)$; $V_{OS} = \Delta V_t$; $V_{OS} = (V_{OV}/2)(\Delta R_D/R_D)$; $V_{OS} = (V_{OV}/2)(\Delta(W/L)/(W/L))$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$; unity gain freq for $T(s) = \dfrac{A_M}{1+(s/\omega_{3dB})}$ for $A_M \gg 1 \Rightarrow \omega_t \simeq |A_M|\omega_{3dB}$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1+s/z_1)(1+s/z_2)\ldots(1+s/z_m)}{(1+s/\omega_1)(1+s/\omega_2)\ldots(1+s/\omega_n)}$
OTC estimate $\omega_H \simeq 1/(\sum \tau_i)$; dominant pole estimate $\omega_H \simeq 1/(\tau_{max})$

Miller: $Z_1 = Z/(1-K)$; $Z_2 = Z/(1-1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/\sqrt{1+V_{db}/V_0}$;
$\omega_t = g_m/(C_{gs}+C_{gd})$; for $C_{gs} \gg C_{gd} \Rightarrow f_t \simeq (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1+A\beta)$; $x_i = (1/(1+A\beta))x_s$; $dA_f/A_f = (1/(1+A\beta))dA/A$; $\omega_{Hf} = \omega_H(1+A\beta)$; $\omega_{Lf} = \omega_L/(1+A\beta)$;
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1+L)) + d/(1+L)$; $Z_{port} = Z_{p^o}((1+L_S)/(1+L_O))$: $PM = \angle L(j\omega_t) + 180$; $GM = -|L(j\omega_{180})|_{db}$;
Pole splitting $\omega'_{p1} \simeq 1/(g_mR_2C_fR_1)$; $\omega'_{p2} \simeq (g_mC_f)/(C_1C_2 + C_f(C_1+C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2$; $Q \leq 0.5 \Rightarrow$ real poles; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A : $\eta = (1/4)(\hat{V}_O/IR_L)(\hat{V}_O/V_{CC})$; Class B : $\eta = (\pi/4)(\hat{V}_O/V_{CC})$; $P_{DN\_max} = V_{CC}^2/(\pi^2R_L)$;
Class AB : $i_ni_p = I_Q^2$; $I_Q = (I_S/\alpha)e^{V_{BB}/(2V_T)}$; $i_n^2 - i_Li_n - I_Q^2 = 0$

2-stage opamp: $\omega_{p1} \simeq (R_1G_{m2}R_2C_c)^{-1}$; $\omega_{p2} = G_{m2}/C_2$; $\omega_z = (C_c(1/G_{m2} - R))^{-1}$;
$SR = I/C_c = \omega_tV_{ov1}$; will not SR limit if $\omega_t\hat{V}_O < SR$